Equitable Apportionment of Maritime Areas Through the Equiratio Method

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1. *Introduction*

According to *Smith* (1982) approximately 376 potential maritime boundaries can be identified world-wide of which, in June 1982, not more than 24% had been negotiated, i.e. signed or ratified. The above figures take into account the fact that under the new Convention on the Law of the Sea coastal States have the possibility of claiming jurisdiction over 200 nautical miles of the water column, sea bed and subsoil adjacent to their coasts.

The effect of this situation will be that in the years to come negotiators will come under increasing pressure to delimit maritime areas of national jurisdiction, especially in the light of improving exploratory, drilling and exploitation techniques regarding resources on the sea bed or in its subsoil.

As was seen in the recent past negotiators in these questions often need assistance of hydrographic surveyors to guide them through the maze of geodetic datums, chart projections, rhumb lines, geodesics and the intricacies of land and sea portrayal on nautical charts. Moreover, the surveyor will generally be asked for advice and to delineate on the chart or charts the delimitation according to the method provisionally agreed upon between Parties to arrive at an equitable apportionment of the maritime area under consideration.

It is mainly for the hydrographic surveyor, who finds himself in such quandary, that this article has been written.

2. *Equity*

In Articles 74 and 83 of the new Convention on the Law of the Sea identical wording has been used regarding the delimitation of the exclusive economic zone and of the continental shelf respectively, between States with opposite or adjacent coasts. The first paragraph of both articles is worded as follows:

> "The delimitation of the exclusive economic zone (continental shelf) between States with opposite or adjacent coasts shall be effected by agreement on the basis of international law, as referred to in Article 38 of the Statute of the International Court of Justice, in order to achieve an equitable solution."
>
> See *United Nations* (1981).

This wording is kept more general than the one that was originally utilized in the Draft Convention one year earlier. See *United Nations* (1980). In 1980 mention was made of certain methods to be employed, such as the use of the median or equidistance line "when appropriate". This reference to certain methodologies has been deleted from the present text of the Articles 74.1 and 83.1 which, consequently, have become less distinct and refer exclusively to the need to achieve a mutually agreed equitable solution.

It is clear that this emphasis exclusively on equity increases the contentiousness of both articles because of the highly subjective nature of the notion of equity. Applicable methods of delimitation, therefore, may also show a wide variety of intended results according to the manner in which the concept of equity is interpreted. A number of circumstances can be recognized which will influence the notion of what may be considered equitable and thus what may be applicable methods of delimitation leading to equitable boundary lines.

In paragraph 101, points C (1) and D (3), of *International Court of Justice* (1969) two such circumstances are mentioned and worded as follows:

C (1) "delimitation is to be effected by agreement in accordance with equitable principles, and taking account of all the relevant circumstances, in such a way as to leave as much as possible to each Party all those parts of the continental shelf that constitute a natural prolongation of its land territory into and under the sea, without encroachment on the natural prolongation of the land territory of the other;"

followed by:

D (3) "the element of a reasonable degree of proportionality, which a delimitation carried out in accordance with equitable principles ought to bring about between the extent of the continental shelf areas appertaining to the coastal State and the length of its coast measured in the general direction of the coast line, account being taken for this purpose of the effects, actual or prospective, of any other continental shelf delimitations between adjacent States in the same region."

This means that equitable principles shall take into account the natural prolongation of a nation's land territory into and under the sea, as well as a certain degree of proportionality between the delimited sea floor area (continental shelf) appertaining to a coastal State and the length of its coast measured in the general direction of its coastline. In paragraph 133, point A (2), of *International Court of Justice* (1982), however, it can be seen that the natural prolongation of the land into the sea will not always enable one to use that phenomenon to achieve a degree of equitable partitioning. In that paragraph it is said:

> 'the area relevant for the delimitation constitutes a single continental shelf as the natural prolongation of the land territory of both Parties, so that in the present case no criterion for delimitation of shelf areas can be derived from the principle of natural prolongation as such;'

Without any claim to exhaustiveness some additional circumstances can be enumerated which – as the case may be – shall be taken into account when an equitable partitioning is pursued, such as:

1. the general configuration of the coasts, e.g. marked changes in direction;
2. the existence of offshore islands, their size, habitation, positions, etc.;
3. the proven existence of offshore mineral resources, especially those near potential boundary lines;
4. the existence of historic rights or fishing zones in the area to be delimited;
5. prior agreements made between Parties concerned, or agreements existing with third Parties in the area under consideration; and
6. the need to safeguard the legitimate rights of third Parties in the area, such as may be the case with shelf-locked countries.

Clearly, no two continental shelf delimitation cases are the same; each has to be judged on its own merits. Mutual agreement on what will constitute an equitable partitioning will be the first step in any pourparlers regarding delimitation negotiations. Once this point is reached it can be expected that the hydrographic surveyor will be consulted about delimitation and delineation procedures to be applied in order to achieve such equity.

3. *Applicable methods of delimitation*

Since the judgment of the *International Court of Justice (1969)* in the North Sea Continental Shelf Cases it has become understood that no single method of delimitation can be obligatory in all circumstances. Essentially, negotiators have quite an arsenal of applicable methods of delimitation at their disposal. The sole hallmark of applicability of any proposed method of delimitation is the agreement between the two negotiating Parties that the proposed method will produce the equitable results envisaged and agreed upon.

One of the best known methods of partitioning is the construction of the equidistance or the median line, depending on the coastal States being adjacent or opposite respectively. This method was already adopted in the 1956 Report of the International Law Commission and as mentioned by *Beazley* (1982) appeared for the first time in the United Nations General Assembly Document A/CN.4/61/Add.1 of 18 May 1953. The method was included in the 1958 Geneva Convention on the Territorial Sea and the Contiguous Zone (Article 12) and the 1958 Convention on the Continental Shelf (Article 6, paragraphs 1 and 2). Median line and equidistance line are described as the boundary line every point of which is equidistant from the nearest points of the base lines from which the breadth of the territorial sea of each State is measured. It should be mentioned here that a median line is equidistant but that the inverse is not true; an equidistance line between two adjacent coastal States is not a median line. In Volume One, pages 230 to 235, *Shalowitz* (1962) gives a clear description of the construction of equidistance lines.

The median and equidistance line, only when agreed upon by both Parties, can be used as an equitable partitioning boundary line, but is not a panacea for all delimitation problems. Eventual less equitable effects, as caused by the application of the equidistance principle, tend to become more obvious farther offshore, so that it is conceivable to divide a boundary line into several parts, for each of which a different method of equitable partitioning may be agreeable. In such a case the landward part of the boundary may well be constructed according to the equidistance principle, whereas farther offshore this will be prohibited and has to be replaced by a method yielding a more equitable result.

The inequitable result to which application of the equidistance principle may lead can best be illustrated by the continental shelf of a small island State B in Fig. 1. It is assumed that the shelf area is partitioned between island State B and coastal State A, which has a fairly straight coast line. In Fig. 1 the situation is depicted in a simplified manner with a straight line as the low-water line of coastal State A, island State B as a mathematical point and the partitioning between them according to the equidistance method. Now this boundary line, being equidistant from the nearest points of the base lines from which the breadth of the territorial sea of each State is measured, coincides with the definition of the parabola, being the locus of a point which moves in such a manner that its distance from a fixed point (the focus = point B) is equal to its distance from a fixed straight line (the directrix = the low-water line of State A). From the figure it follows that farther offshore the island's share of the continental shelf area becomes increasingly disproportionate, which is in conformity with the geometrical fact that the parabola will continuously move further away from its axis.

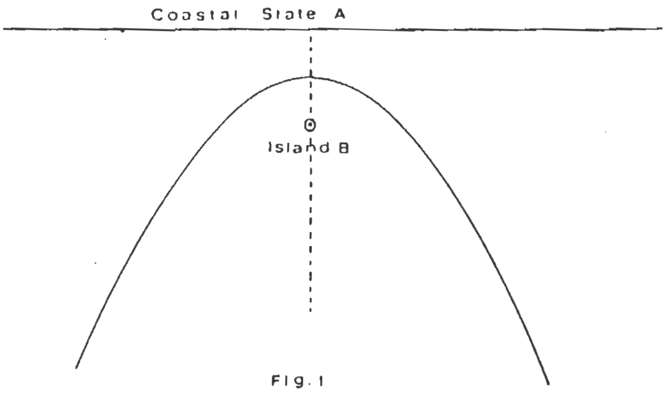

Fig. 1

In the case of two coastal States adjacent to each other along the same straight coast line, an equitable partitioning of the offshore areas could well be achieved through application of the equidistance principle, which would result in a line approximately perpendicular to the low-water line. A well-known complicating factor in such a case is the existence of an offshore island belonging to either of the coastal States. A possible situation is protrayed in Fig. 2, in which are shown two coastal States A and B and an offshore island belonging to State A. Again Fig. 2 shows a simplified picture with a straight low-water line and a point-like island. Assuming that Parties do not object to the application of the equidistance principle, the boundary line is at right angles angles with the low-water line until at point d it meets with the parabola representing the equidistance boundary line between island A and coastal State B. The boundary line between the two coastal States will now follow the parabola in the direction of point e. Unless other circumstances intervene this parabola will be followed until the outer edge of the continental shelf.

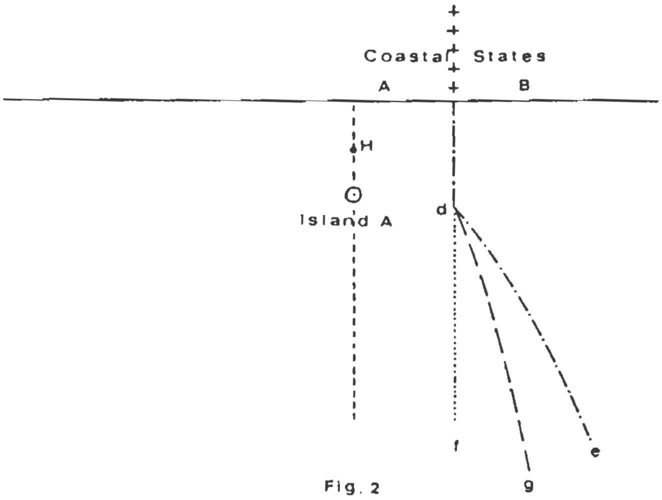

Fig. 2

A much more disproportionate situation would arise if the offshore island, instead of belonging to State A, would belong to coastal State B! This situation is reproduced in Fig. 3. Though it now would be highly implausible that State A might agree to the application of the equidistance principle, this has none the less been assumed in the picture so as to show the reason for State A's reluctance.

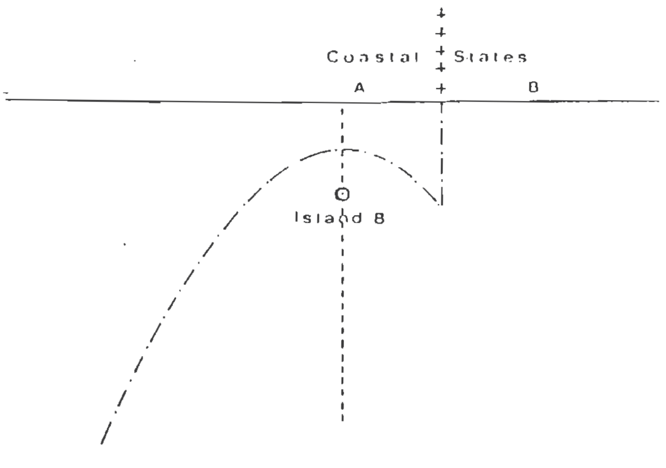

Fig. 3

The picture shows that, because of its geographical position relative to the intersection of the States' border line with the low-water line, the small island B exerts a disproportionally large influence on the course of the boundary line delimiting the offshore areas between the two coastal States, excessively favouring State B.

It is for this reason that some methods were developed to mitigate under cetain circumstances unequitable results produced by the application of the equidistance principle. *Beazley* (1979) in his article on half-effect applied to equidistance lines, gives a number of examples, some theoretically treated, others originating from judgments, such as by the *International Court of Justice* (1982) in the Case Concerning the Continental Shelf (Tunisia/Libyan Arab Jamahiriya).

In Fig. 2 an attempt is made to give half-effect to island A by drawing line d-f, the boundary that would be found if there were no island at all, and consequently construct the curved line d-g all points of which lie half-way between lines d-f and d-e, the latter being the boundary line giving full effect to island A.

Beazley (1979) calls d–e the full-effect line and d–f the no-effect line. He also regards different fictitious situations of island A, such as at point H, half-way between island and low-water line. This would give rise to a different parabola which, however, would not coincide with line d-g, the half-effect line found earlier. Evidently there are different ways to construct so-called half-effect lines which do not altogether coincide. It would, consequently, be better to call such attempts to diminish the effects of equidistant partitioning not 'half-effect' but rather 'partial-effect' methods, which in different guises can be employed by mutual consent. In Beazley's article a number have been discussed.

But even these partial-effect methods cannot under all circumstances provide an acceptable route for part or all of an equidistance line. The picture in Fig. 3 is proof of the difficulty to arrive at any partial-effect method which may acceptably deviate the equidistance boundary line or part of it. It looks as if under certain circumstances it would be better to use not the equidistance method, whether or not adjusted to complicating circumstances, but rather a method enabling a more continuously adjustable delimitation to be achieved. Such a more adaptable method is the so-called "equiratio" method of which the equidistance method can be considered to be a special case.

4. *The equiratio principle of delimitation*

As was expounded above, the equidistance principle, notwithstanding its birthright, will often fail to give satisfaction to negotiating Parties as not leading to equitable results, results which are sometimes difficult to come by, even when mitigating methods are applied to exert their influence on the course of an equidistance boundary line. This is the main reason why some thoughts will be given hereunder to the principle of equiratio, a more general concept than the equidistance one. The former principle can be described as follows.

A boundary line between the offshore areas of two coastal States, either adjacent or opposite, will be called an equiratio line when every point of it will be defined by a constant ratio of its distances from the nearest points of the base lines from which the breadth of the territorial sea of each State is measured. Actual realization of the equiratio principle of delimitation is slightly more complex than is the case with the equidistance one, as the former requires the construction of more curved lines. The rules guiding that construction, however, are clear and simple so that there need be no reason for reluctance to utilize the equiratio principle once negotiating Parties consider it capable of providing an acceptable method of delimitation.

As will be seen hereafter the equiratio boundary line between an offshore (point-like) island State and a straight-shore coastal State, instead of being an ever-expanding parabola as shown in Fig. 1, will have the form of a closed ellipse the size of which will depend on the ratio agreed upon. The advantage of this is evident, as it is now possible to curb the disproportionately increasing continental shelf area appertaining to the island State that would come about by the application of the equidistance principle.

The equiratio boundary line between two opposite coastal States will generally consist of a number of sections all of which will approach circular arcs, unless both States have straight shore-(low-water)lines in which case the equiratio boundary line will also be a straight line. But whatever the geometrical form of the boundary line or sections thereof, the characteristic of the equiratio boundary line is that it lies nearer to one coast than to the other; the degree of inequality in distances being a function of the ratio agreed upon. It is clear that this pliability will enable to take account of very subtle nuances in the appreciation of equity.

In case of adjacent coastal States the equiratio boundary line will generally consist of a number of straight and curved sections, the latter being of the parabolic - and in some cases of the hyperbolic - type. In the equiratio method conic sections play an important role. A conic section is the locus of a point which moves in such a manner that its distance from a fixed point (the focus) bears a constant ratio to its distance from a fixed line (the directrix). If this ratio is equal to unity the curve is called a parabola. If it is less than unity the curve forms an ellipse while it forms a hyperbole when greater than unity.

This article aims both at political negotiators and hydrographic surveyors which latter will generally be asked to give assistance to the former, mainly in the field of delineating on charts the (tentative) methods of delimitation provisionally agreed upon between the negotiators. This means that there are different types of interest involved. The negotiator wants to know what his options are and upon what sort of considerations his possibilities are based. The surveyor, however, needs to know how to perform certain types of delimitation and how to construct on charts the resulting delineating boundary lines.

It is for this reason that the present article from here on will consist of two parts: the article proper and an Appendix. In the appended part information will be given mainly to the hydrographic surveyor on the mathematical background and on how to carry out equiratio delimitations in a number of circumstances. The article proper, therefore, will cater mainly to the negotiator, showing him what possibilities exist under the equiratio principle and to what results they will lead approximately. Of course this part is also of interest to the surveyor and where appropriate reference will be made to relevant sections of the Appendix.

5. *Construction of some equiratio boundary lines around offshore island States*

Starting again with an offshore island State B confronting a coastal State A, as was done in Fig. 1, the need to restrict the delimitation resulting from the application of the equidistance principle is apparent. Applying the equiratio method, however, provides a well-nigh continuous sliding scale enabling the equitable delimitation of the offshore areas however exacting the negotiating Parties may be. The ratio of the surfaces of the offshore areas appertaining to each of the two States can be chosen in such a manner that an equitable solution can always be found.

As was already seen the ellipse will be the locus of all points of which the constant ratio of the distance from a fixed point and from a fixed line is less than unity. In Fig. 4 are shown coastal State A with a straight low-water line and opposing it the island State B represented by a mathematical point. The equidistance boundary line is - like in Fig. 1 - the parabola for which the ratio of the distances is equal to unity. In Fig. 4 this is indicated by the ratio 1/1. Any ratio less than unity - as shown in Fig. 4 - represents an ellipse. For a number of indicated ratios the ellipses have been drawn, either completely or partially. Thus it is seen that for any desired ratio p/q (p<q) an ellipse can be constructed of which the dimensions depend on the values of p and q and on the distance d of island State B from the low-water line of coastal State A. See also App. 1.

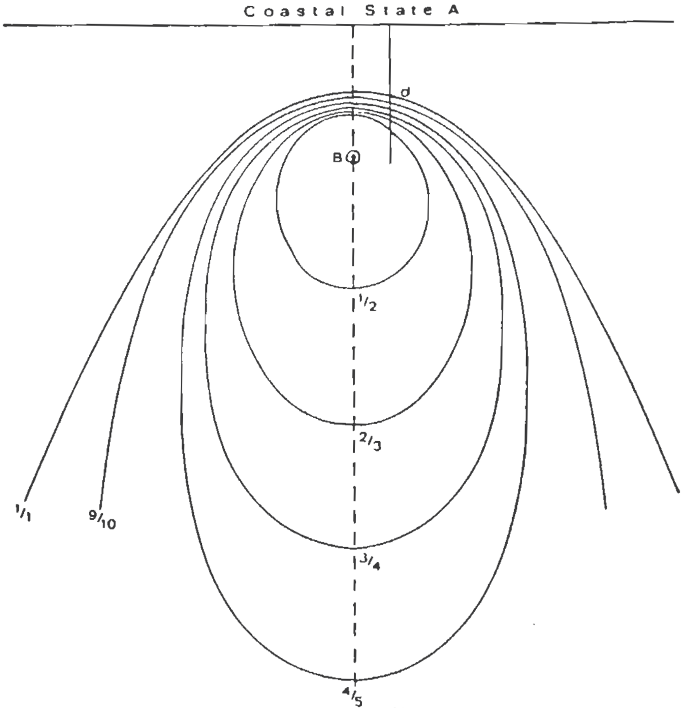

Fig.4

It will, however, not always be possible to consider an island as a mathematical point, without introducing unacceptable distortion. Therefore, a slightly more complex, but still simplified, picture is presented in Fig. 5, showing a triangular island B with straight low-water lines, lying opposite a straight-lined coastal State A

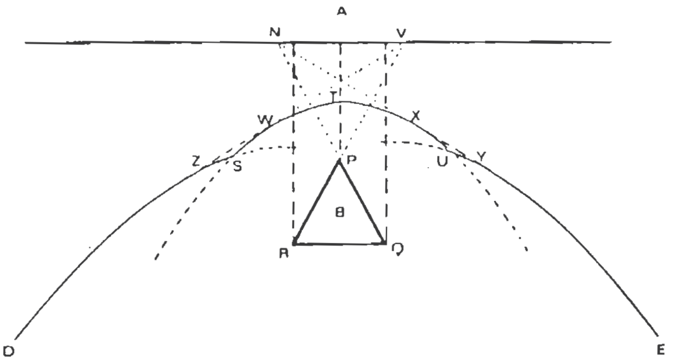

Fig. 5

Now the three points P, Q and R of island B will determine the course of the boundary line between A and B. As is done in the picture the equidistance boundary line has been constructed consisting of several parts. Point P determines parabola STU, whereas at S and U the points of intersection are found with the parabolae DS and UE, determined by the points R and Q respectively.

If the low-water line between P and R is indeed a straight line, then the line ZV would be the equidistance line between the low-water line of the coastal State and line RP so that tangent ZW to both parabolae would become part of the equidistance boundary line TWZD. The same situation may be the case at the east coast of island B which would yield the boundary line TXYE at that side of the island.

Comparison between Fig. 1 and 5 shows that the latter presents an even more disproportionate delimitation, favouring island B, than was the case in the former. In Fig. 6, therefore, a more restricted method of partitioning is shown through the application of the equiratio principle. In Fig. 6 the value of this ratio has been chosen to equal p/q = 3/4. The same method of construction as utilized in Fig. 5 can now be followed in Fig. 6, with the exception that ellipses have replaced the parabolae.

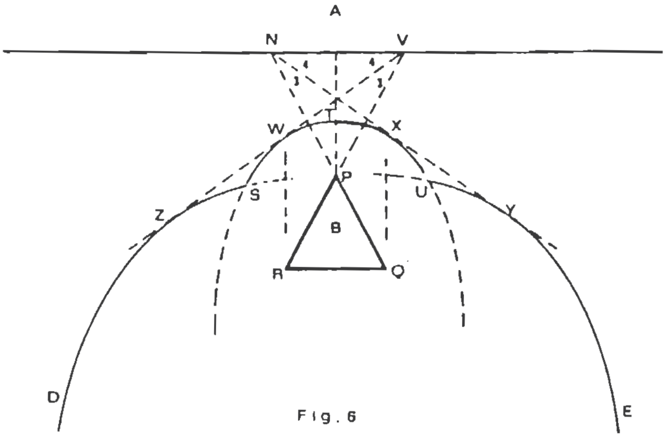

Fig. 6

It will be observed that the common tangents WZ and XY in Fig. 5 remain common tangents in Fig. 6 though in the latter case the common tangents divide the angles NVP and PNV in the ratio 3 : 4, as indicated in the picture. It is obvious that in actual practice simplified situations as have been discussed so far, will seldom occur if at all. The author is convinced, however, that a formalized treatise in which plane coordinate geometrical principles can be applied to simplified geometrical figures, may be of assistance to the surveyor when he has to consider the necessity of applying the more versatile equiratio principle instead of the equidistance one.

6. *Construction of some equiratio boundary lines between adjacent coastal States*

Application of the equidistance principle in the case of adjacent coastal States implies, at least nearest to the low-water line, often the construction of a perpendicular to that line at the point where the land border intersects with the low-water line. In Fig. 7 two adjacent coastal States, P and Q, are shown with a straight low-water line. The border intersects the latter at point L. Perpendicular LS would be the equidistance boundary line for the offshore areas appertaining to P and Q respectively.

Application of the equidistance principle will yield various different delimitation results depending on the ratio agreed upon. In Fig. 7 are drawn lines indicating the distance to the low-water line from 1 to 8 nautical miles. Let us now assume that for one reason or another it is agreed that the distance ratio (dist. to Q) : (dist. to P) = 3 : 4 so that all distances to the nearest points on the low-water line of Q are 3/4 of the distances to the nearest points on the low-water line of P, in accordance with the definition of an equiratio boundary line given at the beginning of paragraph 4. From the picture it can be deduced that in the sector to the right of perpendicular LS the nearest point on the low-water line of State P is point L. Keeping this in mind it becomes clear that in Fig. 7 the line LC represents the line every point of which is 4/3 times as far from point L as from the nearest point on the low-water line of State Q. For construction of line LC as well as any other lines defined by different ratios, the reader is referred to App. 2.

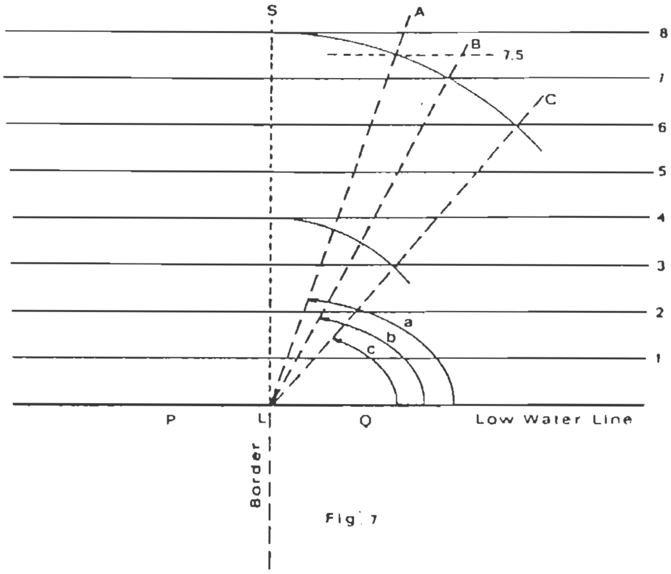

Fig. 7

In Fig. 8 a different situation is represented. The two coastal States P and Q both have straight low-water lines, but at point R, where the land border cuts the low-water line, the latter forms an obtuse angle, thereby producing a concave coast line. Distance lines from 1 to 6 nautical miles have been drawn parallel to the two low-water lines. Line RA, the bisectrix of the obtuse angle at R, is the equidistance boundary line between the offshore areas of P and Q. Application of the equiratio principle will now lead to a different reasoning. It can be seen that in the sector VRU between the perpendicular RU on the one and perpendicular RV on the other low-water line, it is possible to measure the distances to the nearest points on both low-water lines directly, i.e. perpendicularly. According to the definition of an equiratio boundary line, therefore, in this sector such boundary lines are found through the intersection of the relevant distance lines. In this manner is found boundary line RB defined by a ratio 5/6 for the distances to P and Q respectively. Line RB connects point R with the intersection of distance line 6 of Q with distance line 5 of P. Boundary line RB^{1} would have been found had the above ratio been 6/5. In a similar manner boundary line RC has been constructed for a ratio (dist. to P) : (dist. to Q) = 4 : 5. In App. 3 further constructions can be found, esp. outside sector VRU.

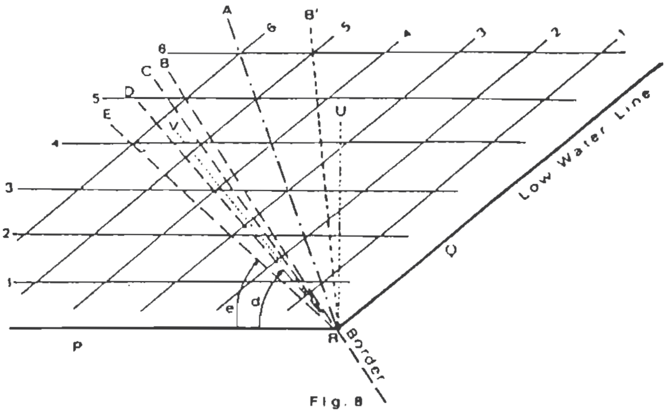

Fig. 8

In Fig. 9 a similar picture is shown as in Fig. 8, with two adjacent coastal States P and Q, both having straight low-water lines and showing a change in direction of the latter at point R. The difference is that in Fig. 9 the land border between States P and Q does not cut the low-water line at the point of change in direction (R), but somewhere else (in S). This situation may have different consequences, depending on the amount of change in direction occurring at point R. In Fig. 9 two different ratios are shown, 7/8 and 3/4.

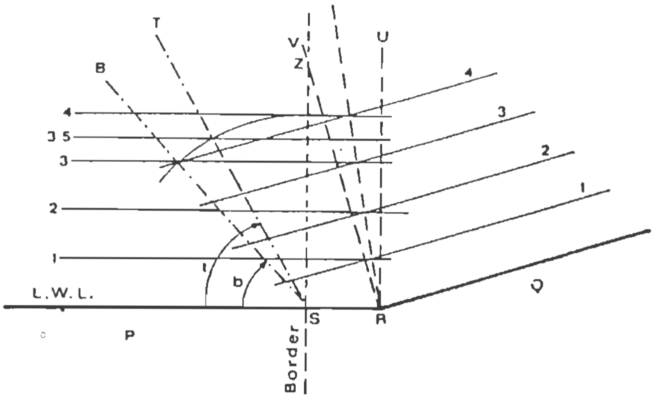

Fig. 9

Construction of the equiratio boundary lines ST and SB respectively is done in the manner indicated in Fig. 7, so that according to App. 2 angle t = 61°.0 and b = 48°.6.

Both boundary lines are diverging with regard to the perpendicular RV, which means that ST as well as SB will not enter the sector between the perpendiculars, sector VRU, and, consequently, will continue to represent the equiratio boundary lines until they cross the outer limit of the offshore area to be delimited.

A similar situation as in Fig. 9 is portrayed in Fig. 10 with the exception that now the angle made by the low-water lines at R is smaller than was the case in Fig. 9 so that now the 7/8 equiratio line SM (angle m = 61°.0) intersects with line RV, the perpendicular to RJ. The equiratio line thereby enters the sector VRU between the two perpendiculars. Within this sector the 7/8 equiratio line is represented by line RT. However, between point S and the point (N) of intersection of RT and SM, the distances to point S of all points lying on the line between S and N, are smaller than to any other point of the low-water line of State Q. It should be remembered that from any point on the line SN the distance to the low-water line of State P equals 7/8 of that to point S (the nearest point of State Q).

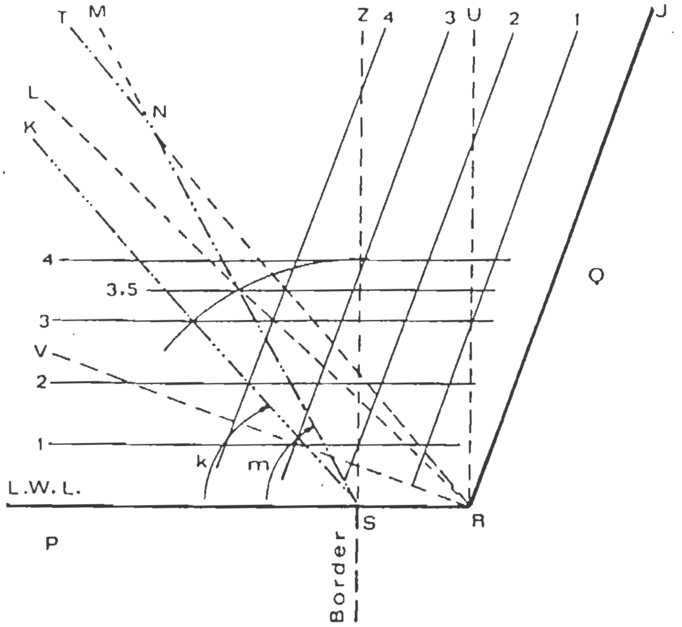

Fig. 10

The point of intersection N is the point where the distance to point S (8/7 times the distance to the low-water line of State P) equals the perpendicular distance to the low-water line of State Q. Consequently, the 7/8 equiratio boundary line between the offshore areas appertaining to coastal States P and Q respectively is represented by the line SNT until the outer limit of the offshore area is reached. It is clear from the picture that the 3/4 lines SK and RL will intersect much farther away.

It should be observed that the angles k and m have constant values, in this case arc sine 0.75 and arc sine 0.875 respectively. This cannot be said of the 7/8, 3/4 or any other equiratio line originating from point R. These lines define angles which depend not only on the ratio concerned but also on the angle the low-water lines make at point R. In App. 4 the mathematical relation existing between p, q and the angle at R will be discussed in more detail.

So far only concave coast lines (angle at $R<180°$) have been considered. The equiratio principle applied to the delimitation of the offshore area of two adjacent coastal States of which the low-water lines change their direction in such a manner that angle $R>180°$, thereby presenting a convex coast line, may develop some additional complications and will, therefore, be discussed later.

7. *Equiratio boundary lines between opposite coastal States*

The situation of two opposite coastal States both with straight low-water lines is simple and application of the equiratio principle will provide again a straight line as the boundary, which will lie nearer to the one than to the other low-water line. The construction of that line is of such simplicity that it need not be discussed here. Moreover, the situation of two opposite straight low-water lines will seldom, if at all, present itself.

Just as is the case with the equidistance median line which normally forms a broken line consisting of parts of perpendicular bisectors, the equiratio line between tw opposite low-water lines will generally represent a broke line consisting of parts of equiratio curves betwee successive pairs of points situated on both low-water line The locus of all points of which the distance from a fixe point bears a constant ratio to its distance from a secon fixed point is a circle of which the centre is situated on th extension of the line connecting the two fixed points and i lying nearest to the fixed point which is nearest to th equiratio line.

In Fig. 11 this situation is shown with the two fixed point at K and M, while N is an arbitrarily chosen point on th circular locus. The situation is comprised in the equatio $NM : NK = p : q$ $(p<q)$ in which p/q is the ratio whic remains constant for all points on the locus.

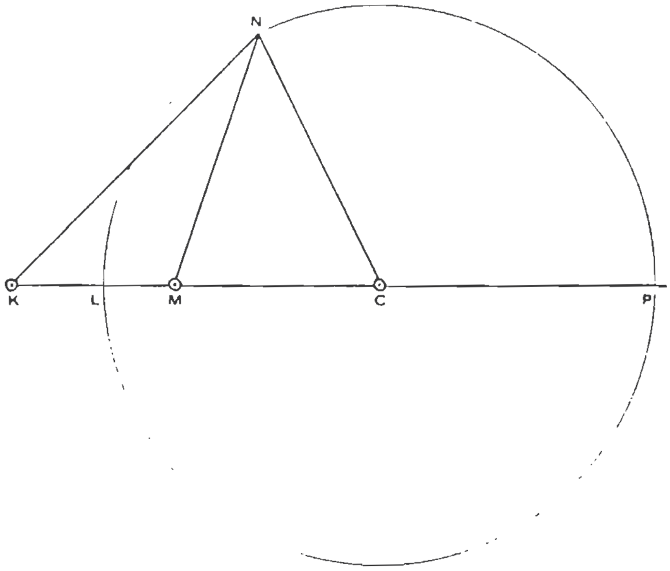

Fig. 11

From the picture it can be deduced that also $LM : LK = p : q$ and if we denote the distance between K and M by the letter 'd' then we may write $LM = p.d/(p+q)$. In App. 5 the mathematical relation between p, q and d on the one hand and the radius of the circle on the other is derived. Thus all elements needed to construct the locus will have become available.

The possibility developed in App. 5 to calculate the radii of curvature of equiratio circles makes it possible to give an impression of the values of these radii for different ratios and a certain distance between the fixed points K and M. For this purpose Fig. 12 was constructed with the aid of (5) and (6) of App. 5. The different centres are indicated by their determining ratios, i.e. 0.5, 0.6, 0.7, 0.75, 0.8, 0.85, 0.9, while for 0.95 the centre falls outside the type area. The radii are drawn to the respective circle arcs which are indicated in the same manner as the centres. The picture provides a nice signpost that the special case of the 1/1 equiratio (=equidistance) line is a straight line, i.e. the perpendicular bisector to the line KM.

Past experience has shown that an equidistance median line between two opposite coastal States consists of a consecutive number of parts of perpendicular bisectors to pairs of relevant points on opposite low-water lines. Furthermore it is a well-known practice to construct such a median line by the trial-and-error method. Exactly the same situation and method of construction is found when the equiratio principle is applied, provided the surveyor takes care to apply different lengths of circle radii to opposite low-

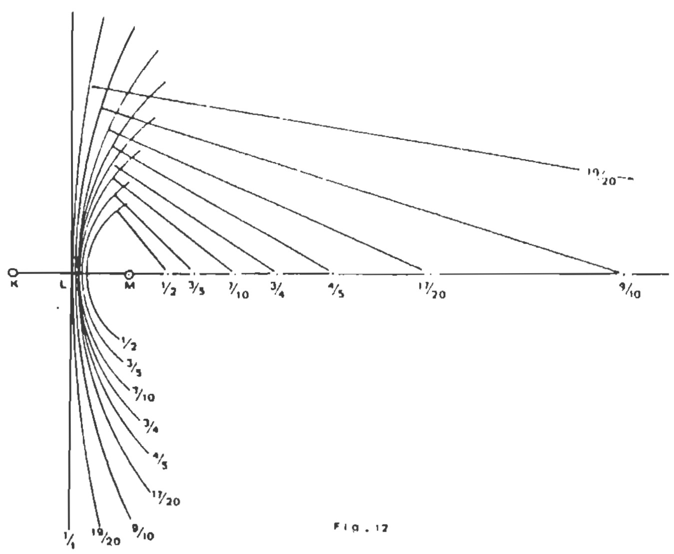

Fig. 12

water lines, in keeping with the ratio agreed upon between negotiating Parties.

An example of such an equiratio boundary line between two opposite coastal States P and Q is shown in Fig. 13 where the ratio $p/q = 3/4$ has been applied. The boundary line runs from Y to Z and passes several circle arcs of which the junctions are indicated by A, B, C, ... to L. In the picture the points of the low-water line of P are denoted by the numerals 1 to 12. Those on the opposite low-water line are numbered 13, 14 and 15. The reader will be able to reconstruct the circle arcs by using (5) and (6) of App. 5, taking into account the different distances between the relevant nearest points on the two low-water lines.

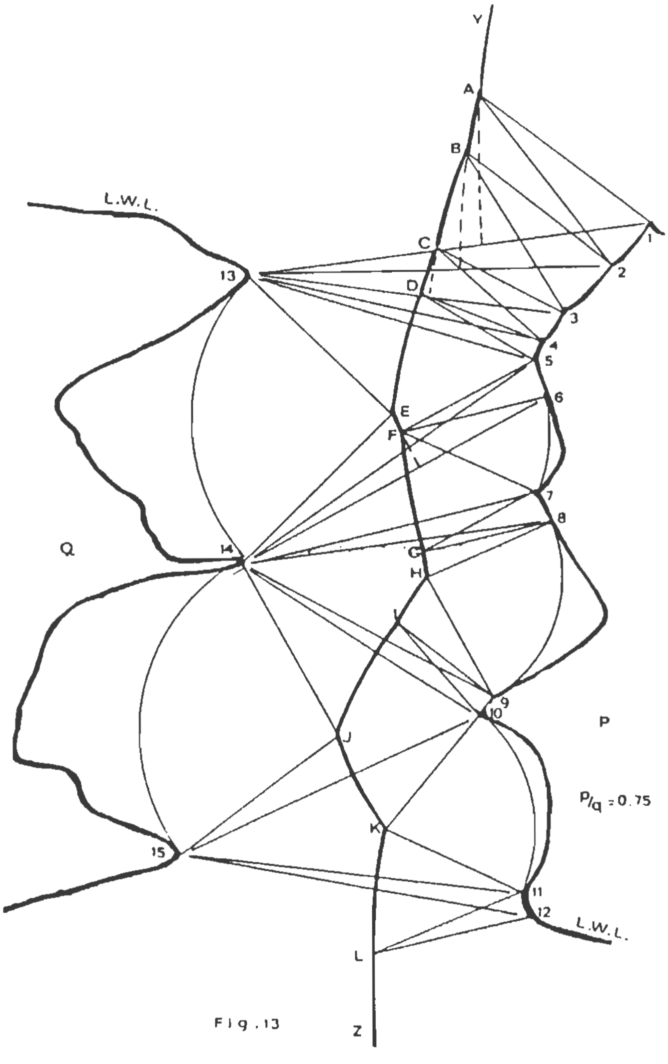

Fig. 13

As (all distances to P) : (all distances to Q) = p : q and as $p<q$, the boundary line runs nearer to P than to Q and it will be understood that all the circle centres lie at the P-side of the boundary line. The above construction is only slightly more complicated than the equidistance one, but opens a nearly continuous scale of possibilities to arrive at a consensus about equitable partitioning.

8. *Equiratio boundary line between two adjacent coastal States situated along a convex coast line*

Before attacking the problem itself, first the remaining conic section has to be discussed as it will make its appearance in the problem of the convex coast line. As was seen earlier the hyperbole is the locus of all points of which the distances from a fixed point bear a constant ratio - which is greater than unity - to their distances from a fixed straight line. The situation is represented in Fig. 14 in which line MY is the fixed straight line and point F is the fixed point. The curve RVZ is part of the hyperbole for which at the vortex V is found VF : VM = q : p ($q>p$) and also RF : RT = q : p. Calling the distance from F to the fixed line, FM = d, then, according to the definition, it is found that $VM = p.d/(p+q)$ and $VF = q.d/(p+q)$. To find the remaining properties of the hyperbole, the reader is referred to App. 6.

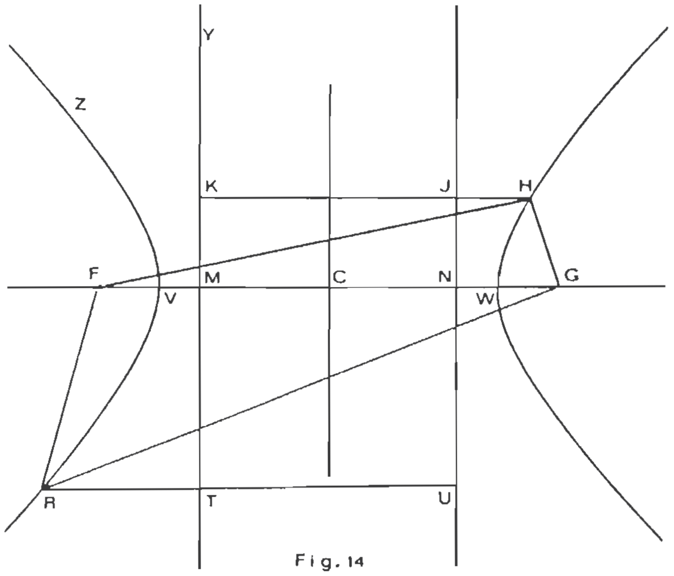

Fig. 14

From App. 6 it follows that all properties needed for the construction of the hyperbole are available, expressed as functions of p, q and d. Consequently, it is now possible to direct our attention at the equiratio boundary line between two adjacent coastal States situated along a convex coast line.

In Fig. 15 again a simplified picture is shown of two straight low-water lines which make an angle of around 250° at point R. The land border intersects with the low-water line at point S. The ratio of distances from the nearest points of the low-water line of State P and of State Q equals $p/q = 3/4$. First the straight boundary line is constructed starting at point S, slanting towards State P at an angle determined by arc sinе 0.75. At point E this offshore boundary line cuts line RU, the perpendicular to the low-water line, and thereby enters into the sector between the two perpendiculars RU and RV. The reader will observe that within this sector URV it is not possible to measure directly the distances to any of the two low-water lines. Within the entire sector the points S and R are the nearest points on the

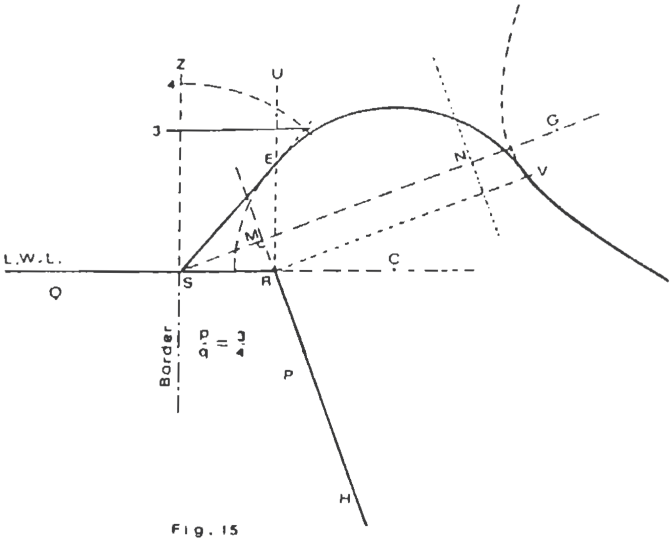

Fig. 15

low-water lines of States Q and P respectively, so that now the 3/4 equiratio locus is defined by the ratio of the distances to these two points (Figs. 11 and 12 refer) and, consequently, is of a circular form. With the aid of (5) and (6) centre point C and the circle arc, EV, are found. At point V the other perpendicular, RV, is crossed and now the circular boundary line leaves the sector URV and enters into the area where the distance to section RH of the low-water line can be measured directly perpendicular to RH.

The locus determining the equiratio boundary line in this area will have to have a constant ratio of the distances from RH and from point S, but the reader will observe that now this ratio equals q/p and, consequently, is greater than unity so that Fig. 14 refers and the locus sought is a hyperbole. However, not the fixed point S (the focus) is now to be utilized, but its reflected image G (the other focus). All parameters to construct the hyperbole can be found with (11) and (12) of App. 6 and the relevant part of the hyperbole will constitute the continuation of the equiratio boundary line; its junction with the circular arc taking place at point V, where the boundary line leaves the sector URV.

In Fig. 15 the ratio utilized was p/q = 0.75. Under certain circumstances this may be considered too keen-edged a reduction of the offshore area appertaining to State P. For this reason Fig. 16 has been added in which the ratio p/q = 0.8 apportions a more liberal part of the offshore area to State P.

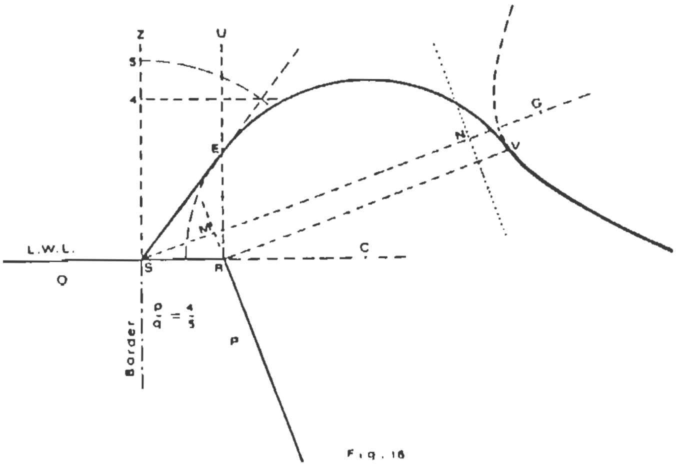

Fig. 16

The reader now has at his disposal all elements needed
find out what a ratio p/q = 7/8 = 0.875 would do to t
delimitation. One thing is certain, however. As long as p/q
less than unity in the configuration shown in Figs. 15 and
the straight-line part of the equiratio boundary line will c
perpendicular RU somewhere, whereafter the boundary li
becomes circular. Whether this part of the boundary li
will be reached, or the hyperbolic part thereafter, depen
on the moment the boundary reaches the outer limit of t
maritime area to be delimitated.

Finally, in order to show the versatility of this equira
principle, Fig. 17 portrays the same geographical situati
as in Figs. 15 and 16, but now the reduction of the ar
offshore appertaining to State P is reduced to p/q = 0.
The reader can see the difference this makes regarding
offshore boundary line; the circular arc, starting at poin
in this case deviates only slowly from the moderat

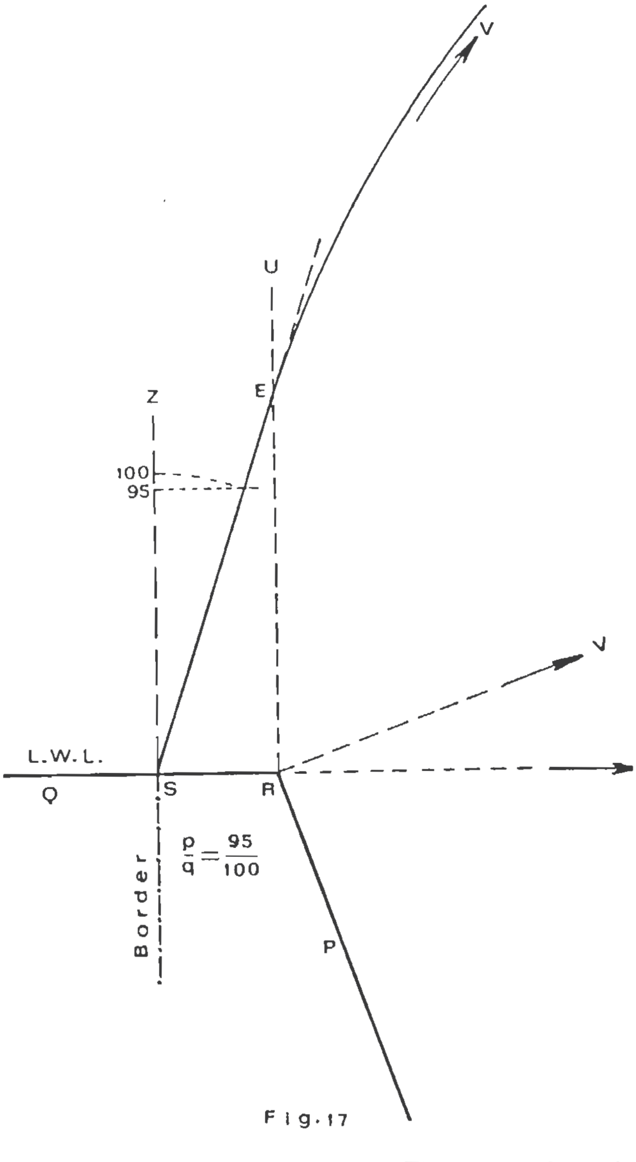

Fig. 17

slanting straight boundary line SE. The centre of the cir
(point C) falls outside the typing area, as well as the poin
where, eventually, the circular arc would change int
hyperbole, provided the boundary line does not cross
outer limit of the offshore area before that point.

9. *Postscript*

The author is well aware of the complexity of many co
lines and low-water lines and the scarce opportunities t

will present themselves to consider a low-water line as a straight line, or an offshore island as a mathematical point for that matter. Still these same irregular features gave birth to the notion of equidistance and allowed of the construction of boundary lines based on that mathematical principle.

For that reason the author is convinced that the equiratio method can be similarly applied, provided negotiating Parties express the wish to do so. As the pliability of the equiratio method, its adaptability to a host of different circumstances, seems greater than that of the equidistance method, whether this latter is mitigated or not by half-effect or partial-effect measures, the chance that the former system will come into use does not seem altogether without foundation.

References

1. *Beazley, P.B.*, 1979: Half-effect applied to equidistance lines. International Hydrographic Review, LVI (1), January 1979: pps 153 - 160: Monaco.
2. *Beazley, P.B.*, 1982: Maritime boundaries. International Hydrographic Review, LIX (1), January 1982: pps 149 - 159: Monaco.
3. *International Court of Justice*, 1969: North Sea Continental Shelf Cases. Judgment of 20 February 1969: 257 pp: The Hague.
4. *International Court of Justice*, 1982: Case Concerning the Continental Shelf (Tunisia/Libyan Arab Jamahiriya). Judgment of 24 February 1982: 323 pp: The Hague.
5. *Shalowitz, A.L.*, 1962: Shore and Sea Boundaries. U.S. Department of Commerce, Coast & Geodetic Survey, Vol. One, 420 pp: Washington D.C.
6. *Smith, R.W.*, 1982: A Geographical Primer to Maritime Boundary Making. Ocean Development and International Law Journal, 12/1-2, 1982 Crane Russak & Company, Inc.: pps 1 - 22.
7. *United Nations*, 1980: Draft Convention on the Law of the Sea. Conference document A/CONF.62/WP 10/Rev.3 (superseded).
8. *United Nations*, 1981: Draft Convention on the Law of the Sea. Conference document A/CONF.62/L.78 ddo 28 August 1981: 175 pp.

Appendices

Appendix 1 (see paragraph 5)

In order to enable the surveyor to construct an ellipse of which the elements p, q and d are known it is necessary to express the ellipse's dimensions in those elements. It is not difficult to see that for every ellipse the apogee (the point farthest away from the low-water line) will lie at a distance $q.d/(q-p)$ from the low-water line. The length of the ellipse's semi-major axis is then found to be equal to $p.q.d/(q^2-p^2)$ and the semi-minor axis will equal $p.d/(q^2-p^2)^{1/2}$. From this it follows that all parameters needed to construct the ellipse can be calculated when p, q and d are known, while factor $q.d/(q-p)$ determines the point of the ellipse farthest away from the low-water line of State A. Especially this latter fact can constitute a major advantage when trying to determine how to achieve an equitable partitioning. In Table 1 the factors with which distance d is to be multiplied have been calculated for a numbr of ratios p/q.

TABLE 1

p/q	$\frac{q}{q-p}$	$\frac{p.q}{q^2-p^2}$	$\frac{p}{\sqrt{q^2-p^2}}$
0.50	2.0000	0.6667	0.5774
0.55	2.2222	0.7885	0.6586
0.60	2.5000	0.9375	0.7500
0.65	2.8571	1.1255	0.8553
0.70	3.3333	1.3725	0.9802
0.75	4.0000	1.7143	1.1339
0.80	5.0000	2.2222	1.3333
0.82	5.5555	2.5031	1.4327
0.84	6.2500	2.8533	1.5481
0.86	7.1429	3.3026	1.6853
0.88	8.3333	3.9007	1.8527
0.90	10.0000	4.7368	2.0647
0.91	11.1111	5.2938	2.1948
0.915	11.7647	5.6213	2.2679
0.92	12.5000	5.9896	2.3474
0.925	13.3333	6.4069	2.4344
0.93	14.2857	6.8838	2.5302
0.935	15.3846	7.4339	2.6364
0.94	16.6667	8.0756	2.7552
0.945	18.1818	8.8338	2.8893
0.95	20.0000	9.7436	3.0424

Appendix 2 (see paragraph 6)

Construction of line LC is accomplished by circling the length of 8 nautical miles, with point L as the centre, until the arc intersects the distance line of 6 nautical miles off the low-water line of State Q. Of course the same line would have been found by circling 4 miles until the 3-mile distance line was cut. It will be clear that angle c, defined by line LC, follows from c = arc sine 3/4 = arc sine 0.75 = 48°.6. In a similar manner have been constructed lines LB and LA, forming the angles b and a respectively. The values of these angles follow from:

b = arc sine 7/8 = arc sine 0.875 = 61°.0 and
a = arc sine 7.5/8 = arc sine 15/16 = arc sine 0.9375 = 69°.6.

It should be observed that a relatively small deviation from equidistance to equiratio may produce a rather considerable divergence of the boundary line compared to the equidistance perpendicular. In the third example above a ratio of 15/16 produces an equiratio boundary line diverging more than 20 degrees from the perpendicular.

Appendix 3 (see paragraph 6)

Though it does not yet make a difference in practice, boundary line RD in Fig. 8 consists of points all of which are nearer to point R than to any other point on the low-water line of State Q, because falling outside the sector VRU. This means that angle d should follow from d = arc sine 3/4 = arc sine 0.75 = 48°.6 and not from the intersection of distance lines 4 and 3 as shown in the diagram. However, the distances measured to the low-water line of Q along the perpendicular or along line DR differ so little (in this case) that it does not exert any practical influence on the value of angle d. This influence increases farther away from perpendicular RV, such as e.g. with regard to angle e. This phenomenon will cause some complications in a situation as depicted in Fig. 9.

Appendix 4 (see paragraph 6)

Referring to Fig. 10 we call the change in direction of the low-water line at R, angle r, and the ratio p/q (q>p), with p relating to State P and q to State Q, then angle TRS, or angle LRS, which will be called angle X, can be calculated from:

$$\tan X = (p/q \sin r) \,/\, (1 + p/q \cos r) \qquad (1)$$

In the case of angle TRS (1) will yield:

$$\tan TRS = (\tfrac{7}{8} \sin 110^\circ)/(1 + \tfrac{7}{8} \cos 110^\circ) = 0.8222 / 0.70075$$
$$= 0.94793 \text{ so that angle } TRS = 49^\circ.6$$

In similar manner angle LRS will then follow from:

$$\tan LRS = (3/4 \sin 110^\circ) \,/\, (1 + 3/4 \cos 110^\circ)$$
$$= 0.70477 \,/\, 0.74348 = 0.94793 \text{ so that angle } LRS = 43^\circ.5.$$

Both these values can be checked in Fig. 10 and give rise to the remark that for a certain ratio the angle, made by the equiratio line through R with the low-water line of the State to which the smaller numerator p appertains, is dependent solely on angle r.

From the Figs. 9 and 10 it becomes clear that only if the equiratio line through R lies within the sector VRU formed by the two perpendiculars to the low-water lines, the possibility exists that somewhere offshore the equiratio boundary line through S will be deflected and, therefore, will follow NT as in Fig. 10. It is, consequently, of importance to know for which value of the angle r the perpendicular RV in Fig. 10 will coincide with the equiratio line through R. In that case (1) will change into:

$$\tan X = (r - 90^\circ) = -\cot r = (\tfrac{p}{q} \sin r)/(1 + \tfrac{p}{q} \cos r) \qquad (2)$$

From (2) can be calculated the value of angle r for which coincidence will take place of RV with the p/q equiratio line through point R. From (2) it follows that

$$-\cot r = p \sin r \,/\, (q + p \cos r) \text{ and}$$
$$-\tan r = \tfrac{q}{p} \operatorname{cosec} r + \cot r \text{ which finally will lead to:}$$
$$\cos r = -p \;\; q \qquad (3)$$

In Table 2 a few values of p/q are given with the associated values of angle r, based on equation (3).

TABLE 2

p/q	cos r	r	p/q	cos r	r
7/8	- 0.875	151°.0	1/3	- 0.333	109°.5
3/4	- 0.750	138°.6	1/4	- 0.250	104°.5
5/8	- 0.625	128°.7	1/5	- 0.200	101°.5
1/2	- 0.500	120°.0	1/6	- 0.1667	99°.6

When angle r is exceeding the value as calculated in Table 2, or rather as defined by (3), then the equiratio line through point R will fall outside the sector VRU between both perpendiculars and then the equiratio line through point S will be the sole boundary line delimiting the offshore areas between States P and Q as represented in Fig. 9.

Appendix 5 (see paragraph 7)
As already followed from Fig. 11 it is possible to write:
$LM = p.d/(p+q)$ and $LK = q.d/(p+q)$ (4)
in which $d = KM$. At point P also is valid:
$PK : PM = q : p$ and $(PK-PM) : (q-p) = PK : q$, from which follows $PK = q.d/(q-p)$ and as $PL = PK - KL$ we finally find, also because of (4): $PL = 2r = q.d/(q-p) - q.d/(p+q)$.
Thus the radius of the locus follows from:
$CN = CL = p.q.d/(q^2-p^2)$ (5)
[illegible], furthermore, $MC = r - LM$, the equations (4) and (5) will yield:
$MC = p.q.d./(q^2-p^2) - p.d/(p+q)$ from which can be derived:
$MC = p^2.d/(q^2-p^2)$ (6)

Equations (5) and (6) contain all information necessary to construct the equiratio circle, once p, q and d are known. In Table 3 the multiplication factors of (5) and (6) have been calculated for a number of ratios p/q.

TABLE 3

p/q	$\frac{p^2}{q^2-p^2}$	$\frac{p.q}{q^2-p^2}$	p/q	$\frac{p^2}{q^2-p^2}$	$\frac{p.q}{q^2-p^2}$
0.60	0.5626	0.9375	0.90	4.2632	4.7368
0.65	0.7316	1.1255	0.91	4.8173	5.2938
0.70	0.9608	1.3725	0.92	5.5104	5.9896
0.75	1.2857	1.7143	0.925	5.9264	6.4069
0.80	1.7778	2.2222	0.93	6.4019	6.8838
0.82	2.0525	2.5031	0.935	6.9507	7.4339
0.84	2.3967	2.8533	0,94	7.5911	8.0756
0.86	2.8402	3.3026	0.945	8.3480	8.8338
0.88	3.4326	3.9007	0.95	9.2564	9.7436

Appendix 6 (see paragraph 8)
From analytical geometry it is known that the hyperbole is symmetrical around a central line through C in Fig. 14, so that the reflected image of vortex V is point W. As was already pointed out in paragraph 8 the distance from point F to the fixed line TY is denoted by $FM = d$ so that, according to the definition, it is found that:
$VM = p.d/(p+q)$ and $VF = q.d/(p+q)$ (7)
For the distance of the reflected vortex W from point M can be found:
$MW = p.d/(q-p)$ (8)
Whereafter it follows from (7) and (8) that
$FW = p.d/(q-p) + d = q.d/(q-p)$ (9)
It also follows from (7) and (8) that
$VW = MW + VM = p.d/(q-p) + p.d/(q+p) = 2.p.q.d/(q^2-p^2)$
The reader will now be able to derive: (10)
$VC = p.q.d/(q^2-p^2)$ and $MC = p^2.d/(q^2-p^2)$ (11)
Finally we find, when G is the reflected image of fixed point F,
$MG = MW + WG = MW + FV = p.d/(q-p) + q.d/(q+p)$
so that

$$MG = \frac{q^2+p^2}{q^2-p^2}.d \qquad (12)$$

It may be worthwhile to remark here that the ratio of e.g.
$FR : RT = q : p = RG : RU$
and that similarly
$HG : HJ = q:p = HF:HK = WG:NW$.

In Table 4 the multiplication factors $p/(q-p)$ and $\frac{q^2+p^2}{q^2-p^2}$ are calculated. The other multiplication factors utilized above can be found in Tables 1 and 3.

TABLE 4

p/q	$\frac{p}{q-p}$	$\frac{q^2+p^2}{q^2-p^2}$	p/q	$\frac{p}{q-p}$	$\frac{q^2+p^2}{q^2-p^2}$	p/q	$\frac{p}{q-p}$	$\frac{q^2+p^2}{q^2-p^2}$
0.45	0.8182	1.5078	0.80	4.0000	4.5556	0.92	11.5000	12.0208
0.50	1.0000	1.6667	0.82	4.5556	5.1050	0.925	12.3333	12.8528
0.55	1.2222	1.8674	0.84	5.2500	5.7935	0.93	13.2857	13.8038
0.60	1.5000	2.1250	0.86	6.1429	6.6805	0.935	14.3846	14.9014
0.65	1.8571	2.4632	0.88	7.3333	7.8652	0.94	15.6667	16.1821
0.70	2.3333	2.9216	0.90	9.0000	9.5263	0.945	17.1818	17.6960
0.75	3.0000	3.5714	0.91	10.1111	10.6347	0.95	19.0000	19.5128